NAIC-ID(RS)T-0627-95

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OPTIMAL DUAL-TANGENTIAL TRANSFER BETWEEN TWO CIRCULAR ORBITS IN CONSIDERATION OF THE EARTH'S FLATTENING EFFECT

by

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# HUMAN TRANSLATION

NAIC-ID(RS)T-0627-95 9 February 1996

MICROFICHE NR: 96C000071

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English pages: 12

Source: "Kao Lyu Di Qiu Pian Lyu Xiao Ying De Liang Yuan Gui Dao Zhi Jian De Shuang Gong Qie Zui You Zhuan Yi"; Astronavigational Control, No. 1, 1994, pp. 13-18

Country of origin: China
Translated by: SCITRAN
F33657-84-D-0165
Requester: NAIC/TASC/Richard A. Peden, Jr.

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# I. INTRODUCTION

With regard to cases which do not give consideration to orbital perturbations, research results associated with optimum transfer problems between two circular orbits have been quite numerous. Among them, the most famous and practically useful are dual tangential plans. The plans in question are designs which, in accordance with dual impulse methods (dual pulse type), make spacecraft realize transfer energies from one circular orbit (the initial orbit) to another circular orbit (the final orbit) most economically. The basic points are described below. First of all, when the spacecraft moves along the initial orbit to the intersection line of the planes of the two circular orbits (first orbital change point), it enters into the first orbital change (applying the first impulse). This causes it to transfer into elliptical orbit (dual tangential transfer orbit) motion with apogees and perigees respectively on the two circular orbits. After that, waiting until when the spacecraft moves along the elliptical transfer orbit to the location of the terminal orbit (Also on the intersection line between the planes of the two circular orbits. This second orbital change point and the first orbital change point are respectively set on the two sides of the center of the earth.), it then enters into the second orbital change (application of the second impulse), making it transfer into terminal orbit motion. The essential elements of the planes of the elliptical orbits in question (that is, angle of inclination and ascending nodal point right ascension) are dependant on the key orbital elements of initial orbits and terminal orbits (radius, angle of inclination, and ascending node right ascension). Specific values are determined in accordance with optimized conditions.

However, the results discussed above are only capable of acting as one type of approximate solution for actual orbital transfer problems. Because the gravitational field of the earth

is certainly not a strictly central field of force, and space close to the earth is not an absolute vaccuum, spacecraft orbits, therefore, always exhibit perturbations. When considering key orbit perturbation factors, optimum transfer problems between two circular orbits--without much more discussion--thereby give more practical value to engineering designs. For this reason, this article draws up--in cases where flattening effects associated with the earth are considered--probes into optimum plans where the greatest energy savings realize--in accordance with dual impulse methods--transfer from one circular orbit to another circular orbit along dual tangential elliptical orbits.

Obviously, when consideration is given to the effects of flattening associated with the earth--besides spacecraft moving in an equatorial plane being able to make circumferential motions around the earth--in other cases, osculating orbits associated with spacecraft motion orbits are not capable of maintaining a round shape. At most, they are only capable of being near circular shapes or osculating orbits which, at certain instants, achieve a round shape. As a result, as far as the circular orbits of this article are concerned, it is only possible to carry out solutions by a type of approximation in accordance with osculating orbits at instants of orbital change being circular shapes as well as with regard to near circular orbits. No further explanation will be made. /14

As to opting for the use of dual tangential transfer orbits as a starting point, on the one hand, it is based on the orbits in question being optimum transfer orbits when orbital perturbations are not figured in. On the other hand, it is possible to achieve simplifications of optimum transfer problems when considerations are made of orbital perturbations. Besides that, only considering flattening effects associated with the earth certainly does not mean that other perturbation factors do not exist. It is just that, in order to simplify discussions of worked out effects or effects believed to be created by other preturbation factors on orbits, it is possible to go through

orbital maintenance systems to provide their elimination. It is also necessary to put forward that, with regard to flattening perturbations associated with the earth, consideration is only given to the two primary effects of orbital plane forward motion and perigee (line of apsides) rotation.

## II. DETERMINATION OF KEY DUAL TANGENTIAL OPTIMAL TRANSFER ORBITAL PLANE FACTORS

If one wants to determine optimal transfer plans to be realized along dual tangential elliptical orbits between two circular orbits in accordance with dual impulse methods, one first of all needs to solve for the key elements associated with the planes of the transfer orbits in question.

Assume that the key orbital elements of the two circular orbits--initial orbit and final orbit--are, respectively as follows.

Initial orbit radius is R1. Angle of inclination is i1. The ascending node right ascension associated with instant t osculating orbits is Ω1.

Final orbit radius is R2. Angle of inclination is i2. The ascending node right ascension associated with instant t osculating orbits is Ω2.

Note that dual tangential elliptical transfer orbit semiparameter is PT. Eccentricity is eT. Angle of inclination is iT. The ascending node right ascension associated with instant t osculation orbits is ΩT. Perigee angular distance (measurement from ascending node to perigee point along the direction of orbital movement) is $\omega_T$.

In accordance with the meaning of dual tangential, it is possible to know that

$$P_T = \frac{2R_1R_2}{R_1 + R_2} \qquad (1)$$

$$e_T = \frac{|R_1 - R_2|}{R_1 + R_2} \qquad (2)$$

In accordance with research results [1,2] associated with pulse type orbit changes, in regard to the two velocity increments $|\Delta \overline{V}_1|$, and $|\Delta \overline{V}_2|$, they are, respectively,

$$|\Delta \overline{V}_1| = V_1 \left\{ 1 + \frac{2R_2}{R_1 + R_2} - 2\left( \frac{2R_2}{R_1 + R_2} \right)^{\frac{1}{2}} \times [\cos i_T \cos i_1 + \sin i_T \sin i_1 \cos(\Omega_{T1} - \Omega_{11})] \right\}^{\frac{1}{2}} \qquad (3)$$

$$|\Delta \overline{V}_2| = V_2 \left\{ 1 + \frac{2R_1}{R_1 + R_2} - 2\left( \frac{2R_1}{R_1 + R_2} \right)^{\frac{1}{2}} \times [\cos i_T \cos i_2 + \sin i_T \sin i_2 \cos(\Omega_{T2} - \Omega_{22})] \right\}^{\frac{1}{2}} \qquad (4)$$

In the equations, V1 and V2, are, respectively, revolving velocities $(V_1 / V_2 = (R_2 / R_1)^{\frac{1}{2}})$ ; ΩT1 and Ω11 are, respectively, the ascending node right ascensions associated with transfer orbits for the first orbital change instant and respective osculating orbits associated with initial orbits. ΩT2 and Ω22 are, respectively, the ascending node right ascensions associated with transfer orbits for the second orbital change instant and respective osculating orbits associated with final orbits. Besides this, note that the ascending node right ascension associated with the first orbital change instant final orbit osculating orbits is Ω21. It is possible to take note of the fact that there is no clear functional relationship between equation (3) and equation (4) and transfer orbit perigee angular distances. However, this is certainly not to say that changes associated with transfer orbit perigee angular distances do not make velocity increments needed for orbital changes (below we will see the influences on $|\Delta \overline{V}_2|$ values) give rise to alterations.

In situations associated with opting for the use of dual tangential transfer plans involving the shortest time periods

(that is, setting out from the first orbital change point along dual tangential transfer orbits and applying second orbital changes when first reaching final orbits), due to perturbation effects associated with movement forward along the planes of relevent orbits given rise to by earth flattening and angular perigee distance alterations, these two will lead to the forms of relationship set out below /15

$$\Omega_{T2} = \Omega_{T1} + t_T \dot{\Omega}_T \quad (5)$$

$$\Omega_{22} = \Omega_{21} + t_T \dot{\Omega}_2 \quad (6)$$

In the equations, $\dot{\Omega}_T$ and $\dot{\Omega}_2$ are, respectively, rates of change associated with ascending nodes of transfer orbits and final orbits. tT is the time gone through in moving along transfer orbits from first orbital change points to second orbital change points.

$$\dot{\Omega}_T = -10\left(\frac{2R_e}{R_1 + R_2}\right)^{3.5} \cos i_T / D \quad ({}^{(o)}/s) \quad (7)$$

$$\dot{\Omega}_2 = -10\left(\frac{R_e}{R_2}\right)^{3.5} \cos i_2 / D \quad ({}^{(o)}/s) \quad (8)$$

In the equations, Re is global equatorial radius; D is mean solar day (using seconds as measurement).

tT requires considering changes in perigee (or apogee) angular distances, that is, half periods when orbital perturbations exist where tT will deviate from transfer orbits. Under conditions where orbital period changes given rise to by flattening earth effects are ignored as well as assuming that surface areas and velocities are still maintained as fixed values

along orbital motions, tT can be calculated in accordance with the equation below (in the vicinities of second orbital change points, circular arcs are taken to replace transfer orbits)

$$2t_T = T_T\left[1-\dot{\omega}_T\frac{R_2^2}{180(R_1+R_2)(R_1R_2)^{\frac{1}{2}}}T_T\right]^{-1} \qquad (9)$$

In the equation, 180 is actually 180 degrees. TT is the period associated with transfer orbits. $\dot{\omega}_T$ is the rate of change associated with transfer orbit perigee (or apogee) angular distances.

$$T_T = 2\pi\left[\frac{(R_1+R_2)^3}{8\mu}\right]^{\frac{1}{2}} \quad (S) \qquad (10)$$

$$\dot{\omega}_T = -5\left(\frac{2R_e}{R_1+R_2}\right)^{3.5}(1-5\cos^2 i_T)/D \quad (^{(o)}/S) \qquad (11)$$

In the equations: $\mu$ is the earth's gravitational constant.

From equation (3) to equation (11), it is possible to see that $|\Delta\vec{V}_1|$ and $|\Delta\vec{V}_2|$ depend on the key transfer orbit plane elements iT and ΩT1. With a requirement that amounts of orbital change energy associated with dual tangential transfers be minimal, iT and ΩT1 should satisfy limit value equations of the forms set out below

$$\frac{\partial f}{\partial i_T} = 0 \qquad (12)$$

$$\frac{\partial f}{\partial \Omega_{T1}} = 0 \qquad (13)$$

In the equations: f is the sum of velocity increments associated with dual orbit changes.

$$f = |\Delta \vec{V}_1| + |\Delta \vec{V}_2| \quad (14)$$

In this way, it is possible in the end to obtain the two fixed solution equations /16

$$\left[\sin i_T \cos i_1 - \cos i_T \sin i_1 \cos(\Omega_{T1} - \Omega_{11})\right] \frac{V_1 R_2}{|\Delta \vec{V}_1| R_1} \quad (15)$$

$$= -\left[\sin i_T \cos i_1 - \cos i_T \sin_{i2} \cos(\Omega_{T2} - \Omega_{22}) + \sin i_T \sin i_2 \sin(\Omega_{T2} - \Omega_{22})(t_T \frac{\partial \dot{\Omega}_T}{\partial i_T}\right.$$

$$\left. + \dot{\Omega}_T \frac{\partial t_T}{\partial i_i} - \dot{\Omega}_2 \frac{\partial t_T}{\partial i_i})\right] \frac{V_2}{|\Delta \vec{V}_2|}$$

$$\sin i_T \sin i_1 \sin(\Omega_{T1} - \Omega_{11}) \frac{V_1 R_2}{|\Delta \vec{V}_1| R_1}$$

$$= -\sin i_T \sin_{i2} \sin(\Omega_{T2} - \Omega_{22}) \frac{V_2}{|\Delta \vec{V}_2|} \quad (16)$$

In the equations:

(17)

$$\frac{\partial \dot{\Omega}_T}{\partial i_T} = 10k\left(\frac{2R_e}{R_1 + R_2}\right)^{3.5} \sin i_T / D(1/s)$$

$$\frac{\partial t_T}{\partial i_T} = \frac{R_2^2 T_T^2}{360(R_1 + R_2)(R_1 R_2)^{\frac{1}{2}}} \left[1 - \dot{\omega}_T \frac{R_2^2 T_T}{180(R_1 + R_2)(R_1 R_2)^{\frac{1}{2}}}\right]^{-2} \frac{\partial \dot{\omega}_T}{\partial i_T} \quad (s/\omega) \quad (18)$$

$$\frac{\partial \dot{\omega}_T}{\partial i_T} = -50k\left(\frac{2R_e}{R_1 + R_2}\right)^{3.5} \sin i_T \cos i_T / D(1/s)$$

(19)

k is the conversion coefficient, equaling 0.0174533 (1/°). This is due to angles in this article all opting for the use of degrees as units of measurement. In equation (18), 360 and 180 are really 360 degrees and 180 degrees.

Making use of equation (3) to equation (11) as well as equation (15) to equation (19), through numerical value solution methods, it is possible to specifically solve for the values of iT and ΩT1 . From this, specific determinations are also then made for key transfer orbit plane elements associated with dual tangential optimal transfer plans. The precise determination of orbit change points is discussed below.

## III. PRECISE DETERMINATION OF DUAL TANGENTIAL OPTIMAL TRANSFER ORBITAL CHANGE POINTS

After solving for key transfer orbit plane elements associated with dual tangential optimal transfer plans, use is made of the first orbital change point, which should be on intersection points of initial orbit and transfer orbit planes, and of second orbital change points, which should be on intersection points of final orbit and transfer orbit planes. It is not difficult to precisely determine their locations [2].

First orbital change points, at angular distances $\theta_1$ , on osculating orbit planes associated with initial orbits for orbit change instant t (within the orbital planes in question, angles measured beginning from ascending nodes along the direction of motion) are determined by the equation below

$$\sin i_T \sin(\Omega_{T1} - \Omega_{11})\cos\theta_1 + \left[\cos i_T \sin i_1 - \sin i_T \cos i_1 \cos(\Omega_{T1} - \Omega_{11})\right]\sin\theta_1 = 0 \qquad (20)$$

Second orbital change points, at angular distances $\theta_2$ , on osculating orbit planes associated with final orbits for orbit change instant t (within the orbital planes in question, angles measured beginning from ascending nodes along the direction of motion) satisfy the equation below

$$\sin i_T \sin(\Omega_{T2} - \Omega_{22})\cos\theta_2 + \left[\cos i_T \sin i_2 - \sin i_T \cos i_2 \cos(\Omega_{T2} - \Omega_{22})\right]\sin\theta_2 = 0 \qquad (21)$$

Making use of equation (15) and equation (16), it is possible to prove that, in situations where orbital perturbations are not considered, first orbital change points are, in fact, located /17 on intersection points between initial orbit and final orbit planes. However, when orbital perturbations are figured in, due to the existence of $\dot{\Omega}_T$ and $\dot{\omega}_T$ , first orbital change points are certainly not generally located right on the intersection points of initial orbit and final orbit osculating orbit planes associated with orbital change instants.

From equation (20), it is possible to know that $\theta_1$ has two values. From equation (21), it can also be seen that $\theta_2$ has two solutions as well. As far as specific selections are concerned, they should work in concert with $\theta_1$ in order to satisfy the requirements of dual tangential transfer orbits. With regard to this, this article does not make a detailed discussion.

At this point, dual tangential optimal transfer plans have been completely determined.

## IV. SPECIAL SOLUTIONS

With regard to the special case where i2 = 0°, from equation (16), it is possible to immediately obtain

$$\Omega_{T1} = \Omega_{11} \qquad (22)$$

Equation (15) simplifies to become

$$\frac{\left(\frac{R_2}{R_1}\right)^{\frac{1}{2}}\sin(i_1 - i_T)}{\left[1+\frac{2R_2}{R_1+R_2}-2\left(\frac{2R_2}{R_1+R_2}\right)^{\frac{1}{2}}\cos(i_1 - i_T)\right]^{\frac{1}{2}}} = \frac{\left(\frac{R_1}{R_2}\right)^{\frac{1}{2}}\sin i_T}{\left[1+\frac{2R_1}{R_1+R_2}-2\left(\frac{2R_1}{R_1+R_2}\right)^{\frac{1}{2}}\cos i_T\right]^{\frac{1}{2}}} \tag{23}$$

Although, as far as equation (22) and equation (23) are concerned, results are the same as when orbital perturbations are not figured in, ΩT2, however, is certainly not the same as ΩT1 .

From equation (20) and equation (22), it is also possible to obtain

$$\sin\theta_1 = 0 \tag{24}$$

that is, $\theta_1$ can be taken as 0° or 180°.

From equation (21), it is possible to obtain (because iT $i_T \neq$ 0°)

$$\sin(\theta_2 - \Omega_{T2} + \Omega_{22}) = 0 \tag{25}$$

Due to first orbital change points being located on equatorial planes, starting out from limit angles, Ω21 can be

taken to be Ω11 (With regard to i2 = 0°, ascending nodes already have no significance. In all cases, relevent parameters are understood from limit angles). As a result, from equation (5) and equation (6), one has

$$\Omega_{T2} - \Omega_{22} = t_T(\dot{\Omega}_T - \dot{\Omega}_2) \tag{26}$$

Taking equation (26) and substituting into equation (25), one gets

$$\sin[\theta_2 - t_T(\dot{\Omega}_T - \dot{\Omega}_2)] = 0 \tag{27}$$

This is nothing else than to say that $\theta_2$ should be chosen from among the two values tT $(\dot{\Omega}_T - \dot{\Omega}_2)$ and 180° tT $(\dot{\Omega}_T - \dot{\Omega}_2)$ in order to work in concert with $\theta_1$ (0° or 180°), making dual tangential conditions achieve adequacy. It should be brought up that $\theta_2$ here is measured using Ω22, calculated in accordance with equation (6), as datum. If one uses Ω21 (that is, Ω11) as measurement datum, angular distances $\theta_2$ associated with /17 second orbital change points can provide selection values as tT $\dot{\Omega}_T$ and 180° + tT $\dot{\Omega}_T$.

## V. CONCLUSIONS

Due to the existence of orbital perturbations, research on optimal transfers between two circular orbits requires considering their influences. When orbital perturbations are not figured in, optimal transfer plans solved for between two circular orbits are only one type of approximate solution to actual problems. When figuring in orbital perturbations given rise to by flattening associated with the earth, this article gives dual pulse type optimal transfer plans carried out along

dual tangential elliptical orbits (using minimal orbital change energies to act as an index of optimizaton). Relevent formulae are capable of supplying preliminary design uses in engineering. In conjunction with this--in regard to solving dual tangential optimized rendezvous problems associated with two spacecraft in different planes (considering the effects of flattening associated with the earth)--there will also be some benefits.

## REFERENCES


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